NACA TN 4249
10590
TECH LIBRARY KAFB, NM
0066819

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4249

A THEORETICAL ANALYSIS OF THE EFFECT OF ENGINE ANGULAR MOMENTUM ON LONGITUDINAL AND DIRECTIONAL STABILITY IN STEADY ROLLING MANEUVERS

By Ordway B. Gates, Jr., and C. H. Woodling

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
April 1958

TECH

TECH LIBRARY KAFB, NM
0066819

.H

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4249

A THEORETICAL ANALYSIS OF THE EFFECT OF ENGINE ANGULAR MOMENTUM ON LONGITUDINAL AND DIRECTIONAL STABILITY IN STEADY ROLLING MANEUVERS[1]

By Ordway B. Gates, Jr., and C. H. Woodling

SUMMARY

The effect of engine momentum on the longitudinal and directional stability of aircraft in steady rolling maneuvers has been investigated. The results presented indicate that the gyroscopic moments produced on the aircraft by a rotating engine in rolling maneuvers can have an appreciable effect on the range of rolling velocities for which longitudinal or directional instability might occur.

INTRODUCTION

The analysis presented in reference 1 of the effect of steady rolling on the longitudinal and directional stability of aircraft was made for the assumption of zero engine momentum; hence, the results presented were independent of the direction of rolling. Some of the present-day aircraft have exhibited different characteristics in left and right rolls which can be attributed to the asymmetric moments produced on the aircraft by the rotating engine. The purpose of this analysis is to present the aircraft equations which include these asymmetric engine gyroscopic moments and to demonstrate the effects of these terms on the divergence boundaries presented in reference 1 for the steady rolling case. The divergence boundaries presented in this paper are for aircraft having static stability.

SYMBOLS

W weight, lb

T total aerodynamic moment, lb-ft

[1]Supersedes recently declassified NACA Research Memorandum L55G05 by Ordway B. Gates, Jr., and C. H. Woodling, 1955.

M_X	rolling moment, lb-ft
M_Y	pitching moment, lb-ft
M_Z	yawing moment, lb-ft
H	angular momentum, slug-ft^2/sec
I_X	moment of inertia about body X-axis, slug-ft^2
I_Y	moment of inertia about body Y-axis, slug-ft^2
I_Z	moment of inertia about body Z-axis, slug-ft^2
I_{XZ}	product of inertia in X,Z-plane (positive when principal X-axis is below body X-axis at nose), slug-ft^2
I_{X_e}	moment of inertia of engine about body X-axis, slug-ft^2
p	rotational velocity about body X-axis, radians/sec
q	rotational velocity about body Y-axis, radians/sec
r	rotational velocity about body Z-axis, radians/sec
ω_e	engine rotational velocity, radians/sec
V	aircraft velocity, ft/sec
u	component of V along X-body axis, ft/sec
v	component of V along Y-body axis, ft/sec
w	component of V along Z-body axis, ft/sec
α	angle of attack of X-body axis, w/u, radians
β	angle of sideslip, v/u, radians
F	total aerodynamic force, lb
F_X, F_Y, F_Z	components of the aerodynamic force along X-, Y-, and Z-axes, lb
l_3, m_3, n_3	direction cosines relating aircraft body axes to earth Z-axis

t time, sec

$M_q = \frac{\partial M_Y}{\partial q}$

$M_\alpha = \frac{\partial M_Y}{\partial \alpha}$

$N_r = \frac{\partial M_Z}{\partial r}$

$N_\beta = \frac{\partial M_Z}{\partial \beta}$

$\omega_\psi = \sqrt{\frac{N_\beta}{I_Z p^2}}$

$\omega_\theta = \sqrt{\frac{-M_\alpha}{I_Y p^2}}$

$\tau = \frac{\omega_e}{p}$

ξ_θ, ξ_ψ ratio of damping to critical damping in pitch and yaw, respectively

a_4, a_3, a_2, a_1, a_0 coefficients of characteristic equation

D differential operator, $d(\ )/dt$

$\frac{D}{Dt}$ total time derivative

A dot over a symbol indicates differentiation with respect to time.

EQUATIONS OF MOTION

The aircraft equations of motion written in vector form are:

$$\left.\begin{aligned} \frac{D}{Dt}(\overline{H}) &= \overline{T} \\ m\frac{D}{Dt}(\overline{V}) &= \overline{F} + \overline{W} \end{aligned}\right\} \tag{1}$$

where $\frac{D}{Dt}$ refers to total differentiation with respect to time, $\overline{H}$ is the angular momentum vector, $\overline{T}$ represents the aerodynamically applied moments, $\overline{V}$ is the aircraft velocity vector, $\overline{F}$ represents the aerodynamically applied forces, and $\overline{W}$ is the weight of the aircraft. A right-handed system of axes is chosen which originates at the center of gravity of the aircraft and which is fixed in the aircraft. The X-axis is assumed to be coincident with the X-axis of the engine and the X,Z-plane is considered to be the plane of symmetry of the aircraft. Also, the mass distribution of the engine is assumed to be symmetrical about the X-axis, and the engine is assumed to be rotating with constant speed. The rotational velocities about the X-, Y-, and Z-axes are p, q, and r, respectively, and the components of $\overline{V}$ in this system of axes are u, v, and w.

For the previous conditions, the vector momentum is given by

$$\overline{H} = \overline{i}\left(I_X p - I_{XZ} r + I_{X_e}\omega_e\right) + \overline{j} I_Y q + \overline{k}\left(I_Z r - I_{XZ} p\right)$$

where $\overline{i}$, $\overline{j}$, and $\overline{k}$ are unit vectors in X-, Y-, and Z-directions; I_X, I_Y, and I_Z are the moments of inertia; I_{XZ} is the product of inertia in the XZ-plane; I_{X_e} is the moment of inertia of the engine about the X-axis; and ω_e is the rotational velocity of the engine about this axis, taken positive in the same sense as the rolling velocity p.

Equations (1) become, after differentiation and resolution into X-, Y-, and Z-components:

Rolling:

$$I_X\dot{p} - I_{XZ}\dot{r} - \left(I_Y - I_Z\right)qr - I_{XZ}pq = \sum M_X \tag{2a}$$

Pitching:

$$I_Y\dot{q} - \left(I_Z - I_X\right)pr + I_{XZ}\left(p^2 - r^2\right) + I_{X_e}\omega_e r = \sum M_Y \tag{2b}$$

Yawing:

$$I_Z\dot{r} - \left(I_X - I_Y\right)pq - I_{XZ}\dot{p} + I_{XZ}qr - I_{X_e}\omega_e q = \sum M_Z \qquad (2c)$$

X-force:

$$m(\dot{u} + qw - vr) = \sum F_X + Wl_3 \qquad (2d)$$

Y-force:

$$m(\dot{v} - pw + ur) = \sum F_Y + Wm_3 \qquad (2e)$$

Z-force:

$$m(\dot{w} + pv - uq) = \sum F_Z + Wn_3 \qquad (2f)$$

The terms l_3, m_3, and n_3 are the direction cosines between the earth Z-axis and the axes being used, which are fixed in the body. The equations which relate l_3, m_3, and n_3 to the airplane rotational velocities p, q, and r are

$$\dot{l}_3 = m_3 r - n_3 q$$

$$\dot{m}_3 = n_3 p - l_3 r$$

$$\dot{n}_3 = l_3 q - m_3 p$$

Certain assumptions beyond that of a constant rolling velocity are necessary in order to linearize these equations. The term $p^2 - r^2$ in the pitching equation (2b) is taken as approximately equal to p^2; the term $I_{XZ}qr$ in the yawing equation (2c) is considered negligible; and it is assumed that no change occurs in the X-component of the forward velocity. Further, it is assumed that

$$\frac{v}{u} \approx \beta \qquad \frac{w}{u} \approx \alpha$$

and, for the assumption of small out-of-trim aerodynamic forces, the Y-force and Z-force equations (2e) and (2f) are approximately

$$\dot{\beta} = p\alpha - r$$

$$\dot{\alpha} = q - p\beta$$

Also, the aerodynamic moments are taken as

$$M_Y = M_q q + M_\alpha \Delta\alpha$$

$$M_Z = N_r r + N_\beta \beta$$

Equations (2), for these assumptions and substitutions, become in determinant form

$$\begin{array}{ccccc}
q & \Delta\alpha & \beta & r & \\
D - \dfrac{M_q}{I_Y} & -\dfrac{M_\alpha}{I_Y} & 0 & \dfrac{\left(I_X - I_Z\right)p_o + I_{X_e}\omega_e}{I_Y} & = -\dfrac{I_{XZ}}{I_Y} p_o^2 \\
\dfrac{-\left(I_X - I_Y\right)p_o - I_{X_e}\omega_e}{I_Z} & 0 & -\dfrac{N_\beta}{I_Z} & D - \dfrac{N_r}{I_Z} & = 0 \\
0 & -p_o & D & 1 & = p_o\alpha_o \\
-1 & D & p_o & 0 & = 0
\end{array}$$

where p_o refers to a constant value of the rolling velocity p and α_o, to the initial value of the angle of attack. Expansion of this determinant with the right-hand side set equal to zero yields a characteristic equation of the form

$$a_4D^4 + a_3D^3 + a_2D^2 + a_1D + a_0 = 0 \qquad (3)$$

where

$$a_4 = 1$$

$$a_3 = -\frac{N_r}{I_Z} - \frac{M_q}{I_Y}$$

$$a_2 = p_o^2 + \frac{N_\beta}{I_Z} - \frac{M_\alpha}{I_Y} + \frac{M_q N_r}{I_Y I_Z} - \left[\frac{\left(I_Z - I_X\right)p_o - I_{X_e}\omega_e}{I_Y}\right]\left[\frac{\left(I_X - I_Y\right)p_o + I_{X_e}\omega_e}{I_Z}\right]$$

$$a_1 = -\frac{N_r}{I_Z}p_o^2 - \frac{M_q}{I_Y}p_o^2 - \frac{M_q N_\beta}{I_Y I_Z} + \frac{M_\alpha N_r}{I_Y I_Z}$$

$$a_0 = \frac{M_q N_r}{I_Y I_Z}p_o^2 - p_o\frac{N_\beta}{I_Z}\left[\frac{\left(I_Z - I_X\right)p_o - I_{X_e}\omega_e}{I_Y}\right] - \frac{M_\alpha N_\beta}{I_Y I_Z} - p_o\frac{M_\alpha}{I_Y}\left[\frac{\left(I_X - I_Y\right)p_o + I_{X_e}\omega_e}{I_Z}\right] - p_o^2\left[\frac{\left(I_Z - I_X\right)p_o - I_{X_e}\omega_e}{I_Y}\right]\left[\frac{\left(I_X - I_Y\right)p_o + I_{X_e}\omega_e}{I_Z}\right]$$

If the following substitutions are made

$$\omega_\psi^2 = \frac{N_\beta}{I_Z p_o^2}$$

$$\omega_\theta^2 = \frac{-M_\alpha}{I_Y p_o^2}$$

$$2\xi_\theta\omega_\theta = \frac{-M_q}{I_Y p_o}$$

$$2\xi_\psi\omega_\psi = \frac{-N_r}{I_Z p_o}$$

$$\tau = \frac{\omega_e}{p_o}$$

these coefficients become

$$a_4 = 1$$

$$a_3 = p_o\left(2\xi_\psi\omega_\psi + 2\xi_\theta\omega_\theta\right)$$

$$a_2 = p_o^2\left[1 + \omega_\psi^2 + \omega_\theta^2 - \frac{\left(I_Z - I_X - I_{X_e}\tau\right)\left(I_X - I_Y + I_{X_e}\tau\right)}{I_Y I_Z} + 4\xi_\theta\xi_\psi\omega_\theta\omega_\psi\right]$$

$$a_1 = p_o^3\left(2\xi_\psi\omega_\psi + 2\xi_\theta\omega_\theta + 2\xi_\theta\omega_\theta\omega_\psi^2 + 2\xi_\psi\omega_\psi\omega_\theta^2\right)$$

$$a_0 = p_o^4\left[4\xi_\theta\xi_\psi\omega_\theta\omega_\psi - \omega_\psi^2\frac{\left(I_Z - I_X - I_{X_e}\tau\right)}{I_Y} + \omega_\psi^2\omega_\theta^2 + \right.$$
$$\left.\omega_\theta^2\left(\frac{I_X - I_Y + I_{X_e}\tau}{I_Z}\right) - \left(\frac{I_Z - I_X - I_{X_e}\tau}{I_Y}\right)\left(\frac{I_X - I_Y + I_{X_e}\tau}{I_Z}\right)\right]$$

which are essentially equivalent to the coefficients presented on page 9 of reference 1, with the exception of the engine momentum terms. It will be noted that the p_o factors which multiply the coefficients shown here do not appear in reference 1. This difference is attributable to the fact that the differential operator used in this paper is a time operator, whereas in reference 1 the operator has been made a function of rolling velocity.

ANALYSIS AND DISCUSSION

The combinations of ω_ψ and ω_θ that result in a negative value for the constant term a_o of equation (3) for given values of ξ_θ, ξ_ψ, and τ and hence give aperiodic instability are of primary interest. For an airplane with given values of natural frequency in pitch and yaw, these values of ω_ψ and ω_θ define the rolling velocities for which this instability will exist.

A sample of these aperiodic divergence boundaries is shown in figure 1 for $\tau = 0$ $(\omega_e = 0)$ for $\xi_\theta \xi_\psi = 0$ and 0.0031. The mass and aerodynamic characteristics of the airplane for which these boundaries were constructed are presented in table I. It should be noted that boundaries constructed for a constant value of $\xi_\theta \xi_\psi$ do not correspond to a constant value of $\frac{M_q N_r}{I_Y I_Z}$; instead, every point on the curve represents, dimensionally, a different value of this parameter. In this plane, the frequencies in pitch and yaw of a given airplane, for all values of p_o, lie along a straight line similar to the one shown in the figure. The point shown for $p_o = 1$ radian/sec defines the frequencies of the airplane chosen for this illustration. The slope of this line is determined from the ratio of the square of the natural frequencies in pitch and yaw. For the case shown, the frequency locus of this airplane passes through the divergence boundary constructed for $\xi_\theta \xi_\psi = 0.0031$ for $p_o = \pm 1.8$ radians/sec and remains on the unstable side of the boundary up to $p_o = \pm 2.3$ radians/sec. Generally, the characteristic roots of the system in the unstable region of this plane are a pair of stable complex roots, one stable real root, and one unstable real root.

It would be possible to take into account the engine momentum by plotting boundaries for various values of $I_{X_e}\tau$, but, as was mentioned in the discussion of $\xi_\psi \xi_\theta$, these boundaries would not correspond to a constant value of engine momentum. Also, both positive and negative values of $I_{X_e}\tau$ would have to be considered in order to cover both the left and right rolling conditions. The former difficulty can be avoided for both these cases by plotting the boundaries as a function of the dimensional frequency parameters rather than in terms of ω_ψ^2 and ω_θ^2. Presentation in this form necessitates the construction of a boundary for each rolling velocity, but this construction is relatively simple.

A sample of these boundaries is shown in figure 2 for $\frac{M_q N_r}{I_Y I_Z} = 0.044$ and $I_{X_e}\omega_e = 0$. For this case the boundaries for left and right rolling are identical. The effect of the engine momentum on these boundaries can be seen in figure 3 for the case of $I_{X_e}\omega_e = 17,554$ slug-ft^2/sec. Boundaries are presented for both right and left rolls, and the critical roll velocities are shown on the figure for both rolling conditions. For positive (right) rolling the unstable range of p is between $p_o = 2.1$ radians/sec and $p_o = 2.5$ radians/sec and for negative (left) rolling this range is between $p_o = -1.7$ radians/sec and $p_o = -2.2$ radians/sec. In order to specify the absolute range of the roll rate which might be critical for a given airplane, it is necessary to know the critical values of p for both the left and right rolls. For the value of $I_{X_e}\omega_e$ considered here, this unstable range for the example airplane would be defined as being

$$1.7 < |p_o| < 2.5$$

Curves are plotted in figure 4 to show the effect of engine momentum on the values of critical p (p both negative and positive) for the particular airplane being considered. It can be seen that, for $I_{X_e}\omega_e = 0$, the absolute range of critical p is between $|p|$ of 1.8 and 2.3 radians per second and increases to $|p|$ of 1.5 and 2.7 radians per second for $I_{X_e}\omega_e = 40,000$ slug-ft^2/sec. Thus, the range of rolling velocities for which a given airplane might experience instability, based on this steady rolling assumption, can increase appreciably with the magnitude of the momentum of the rotating engine; hence, the effect of engine momentum should be considered in the analysis.

A point of interest with respect to the construction of the divergence boundaries in the dimensional frequency plane is that it is possible to obtain mathematically the envelope of these boundaries by plotting the locus of the points of maximum curvature of the respective curves. The mathematical expression for the curvature of a given function can be found in any calculus text book (for example, ref. 2), and in order to obtain the desired envelope it is necessary only to maximize this expression in the proper manner. From this envelope and the zero and asymptotic values of $\frac{N_\beta}{I_Z}$ and $\frac{-M_\alpha}{I_Y}$, which are rather easy to calculate, the divergence boundaries for any given value of roll rate for specified inertia characteristics can be approximated with reasonable accuracy. In any event, the envelopes of the branches of the curves will define, for the steady rolling case, the combinations of pitch and yaw frequency for which there will be no roll-induced instability. The equations from which the envelopes can be calculated, and expressions for the previously mentioned asymptotic values of $\frac{N_\beta}{I_Z}$ and $\frac{-M_\alpha}{I_Y}$ are:

(a) Equations for determination of zero and asymptotic values of $\frac{N_\beta}{I_Z}$ and $\frac{M_\alpha}{I_Y}$:

$$\left(\frac{M_\alpha}{I_Y}\right)_{\frac{N_\beta}{I_Z}=\infty} = \frac{p_o I_{X_e}\omega_e - \left(I_Z - I_X\right)p_o^2}{I_Y}$$

$$\left(\frac{M_\alpha}{I_Y}\right)_{\frac{N_\beta}{I_Z}=0} = \frac{\frac{M_q N_r}{I_Y I_Z} p_o^2}{\frac{I_X - I_Y}{I_Z}p_o^2 + \frac{I_{X_e}\omega_e p_o}{I_Z}} + \frac{I_{X_e}\omega_e p_o}{I_Y} - \frac{\left(I_Z - I_X\right)p_o^2}{I_Y}$$

$$\left(\frac{N_\beta}{I_Z}\right)_{\frac{M_\alpha}{I_Y}=\infty} = -\left[\frac{\left(I_X - I_Y\right)p_o^2 + I_{X_e}\omega_e p_o}{I_Z}\right]$$

$$\left(\frac{N_\beta}{I_Z}\right)_{\frac{M_\alpha}{I_Y}=0} = \frac{-\frac{M_q N_r}{I_Y I_Z} p_o^2}{\frac{I_{X_e}\omega_e p_o}{I_Y} - \frac{\left(I_Z - I_X\right)p_o^2}{I_Y}} - \left(\frac{I_X - I_Y}{I_Z}\right)p_o^2 - \frac{I_{X_e}\omega_e p_o}{I_Z}$$

(b) Equations for determination of envelope of divergence boundaries:

$$\frac{N_\beta}{I_Z} = -\left[\frac{\left(I_X - I_Y\right)p_o^2 + I_{X_e}\omega_e p_o}{I_Z}\right] \pm \sqrt{\frac{M_q N_r}{I_Y I_Z} p_o^2} = a \pm b$$

$$-\frac{M_\alpha}{I_Y} = -\left[\frac{I_{X_e}\omega_e p_o - \left(I_Z - I_X\right)p_o^2}{I_Y}\right] \mp \sqrt{\frac{M_q N_r}{I_Y I_Z} p_o^2} = c \mp b$$

The equations presented for determination of the envelopes of the divergence boundaries require some further explanation. The combinations of $\frac{N_\beta}{I_Z}$ and $\frac{-M_\alpha}{I_Y}$ which define the envelopes are

$$\frac{N_\beta}{I_Z} = a + b$$

$$-\frac{M_\alpha}{I_Y} = c - b$$

and

$$\frac{N_\beta}{I_Z} = a - b$$

$$-\frac{M_\alpha}{I_Y} = c + b$$

The branch of the family of divergence boundaries to which each combination applies depends on the sign of the rolling velocity. A sample of the envelopes calculated for the boundaries of figures 2 and 3 is shown in figure 5.

CONCLUDING REMARKS

An analysis has been presented to examine the effects of engine momentum on the longitudinal and directional stability of aircraft. The results indicate that the ranges of rolling velocity for which the aircraft might experience a roll-induced aperiodic divergence in steady rolling maneuvers can be appreciably increased by the engine momentum. For a particular airplane used in a sample calculation, the range of critical rolling velocity p was calculated to be, when engine momentum was assumed zero,

$$1.8 < |p| < 2.3$$

and, when an engine momentum of 17,554 slug-ft^2/sec was considered, the range was extended to

$$1.7 < |p| < 2.5$$

For values of engine momentum higher than that assumed for this illustrative example, the range would, of course, be further expanded.

Calculations to show the effects of including engine momentum on the construction of the divergence boundaries were also presented, and an alternate method of construction to that presented in NACA Technical Note 1627 was discussed.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 30, 1955.

REFERENCES

1. Phillips, William H.: Effect of Steady Rolling on Longitudinal and Directional Stability. NACA TN 1627, 1948.

2. Granville, William Anthony, Smith, Percey F., and Longley, William Raymond: Elements of the Differential and Integral Calculus. Rev. ed., Ginn and Co., 1941.

TABLE I.- MASS AND AERODYNAMIC CHARACTERISTICS

OF EXAMPLE AIRPLANE

I_X, slug-ft^2	10,976
I_Y, slug-ft^2	57,100
I_Z, slug-ft^2	64,975
I_{XZ}, slug-ft^2	942
$I_{X_e}\omega_e$, slug-ft^2/sec	17,554
$\frac{N_r}{I_Z}$, $\frac{1}{\text{sec}}$	-0.105
$\frac{M_q}{I_Y}$, $\frac{1}{\text{sec}}$	-0.421
$\frac{M_\alpha}{I_Y}$, $\frac{1}{\text{sec}^2}$	-5.30
$\frac{N_\beta}{I_Z}$, $\frac{1}{\text{sec}^2}$	2.38
V, ft/sec	691

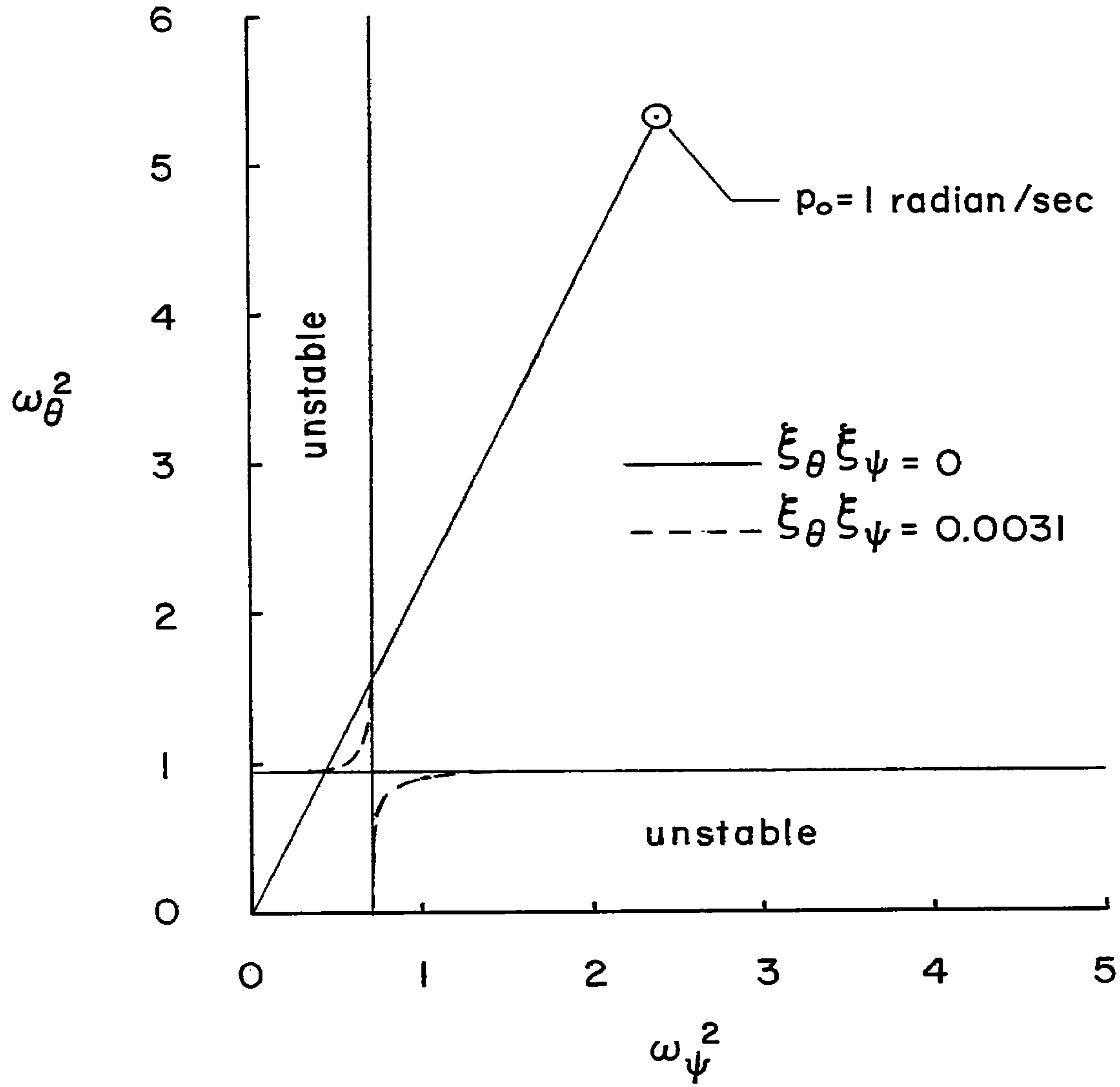


Figure 1.- Boundaries in the ω_ψ^2, ω_θ^2 plane which define regions of aperiodic divergence for example aircraft. $I_{X_e}\omega_e = 0$.

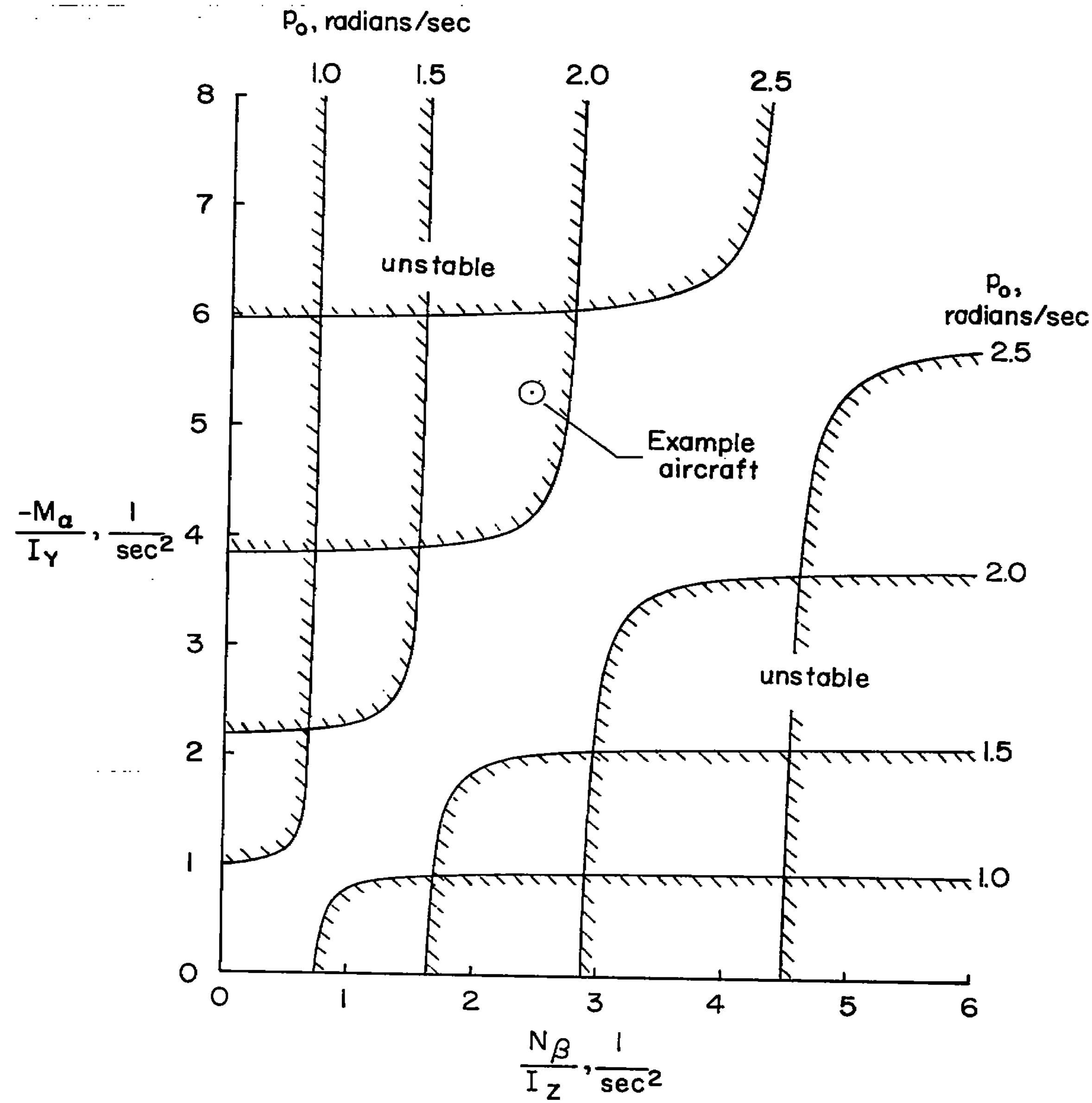
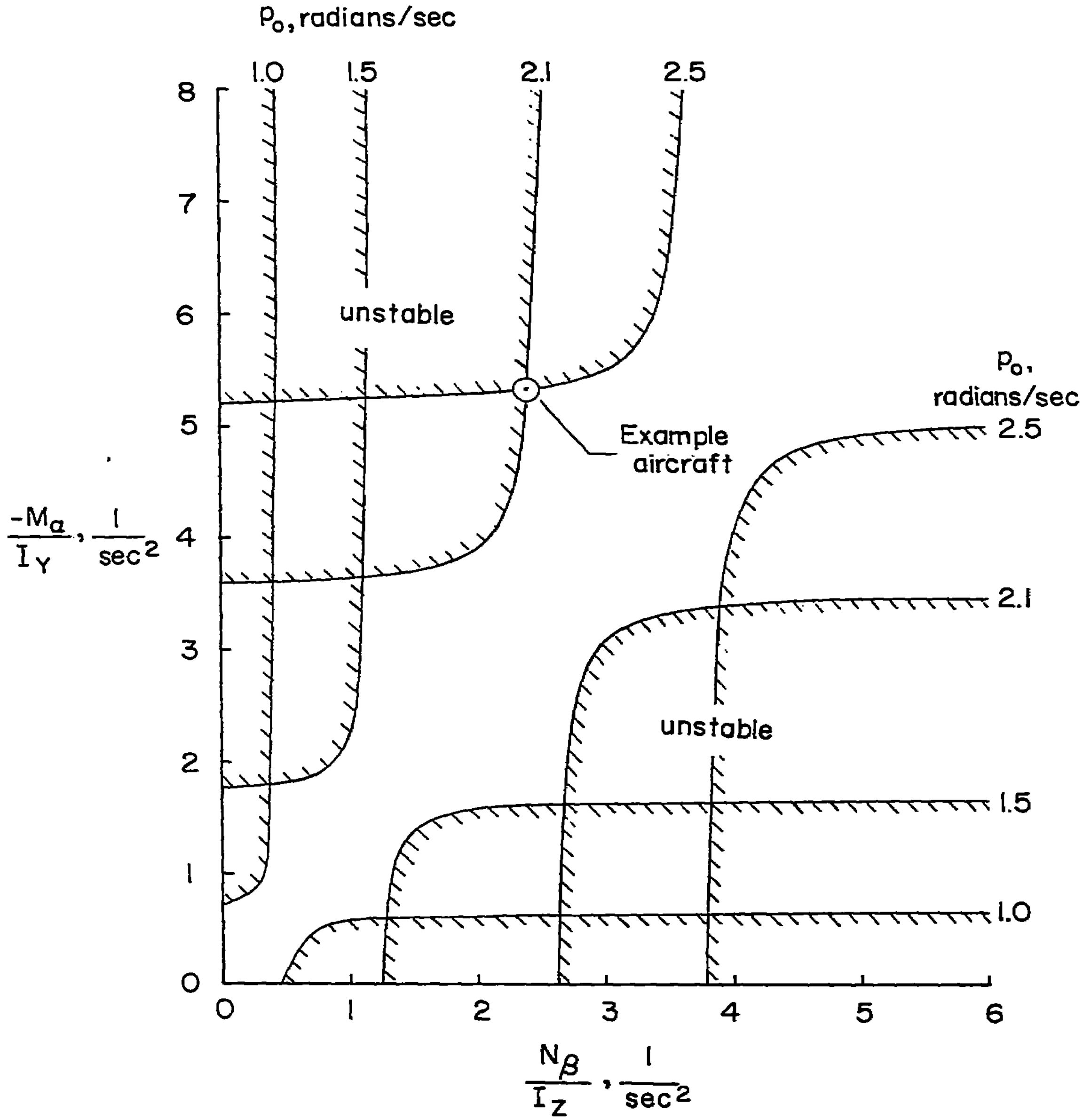
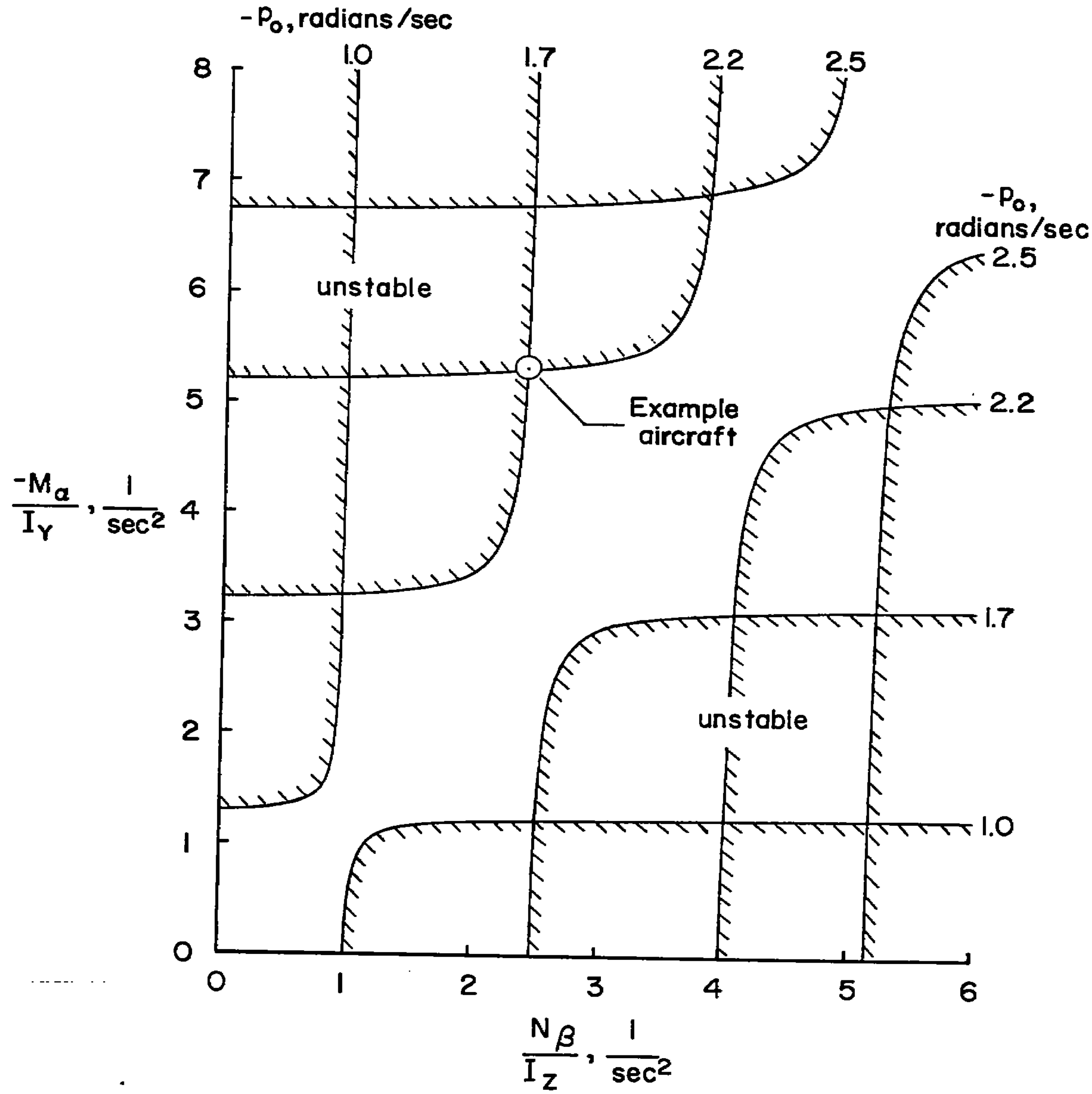


Figure 2.- Boundaries in the N_β/I_Z, M_α/I_Y plane for example aircraft which define regions of aperiodic divergence as a function of rolling velocity. $I_{X_e}\omega_e = 0$; $\frac{M_q N_r}{I_Y I_Z} = 0.044$.



(a) Right rolls.

Figure 3.- Boundaries in the N_β/I_Z, M_α/I_Y plane for example aircraft which define regions of aperiodic divergence as a function of rolling velocity. $I_{X_e}\omega_e = 17{,}554 \frac{\text{slug-ft}^2}{\text{sec}}$; $\frac{M_q N_r}{I_Y I_Z} = 0.044$.



(b) Left rolls.

Figure 3.- Concluded.

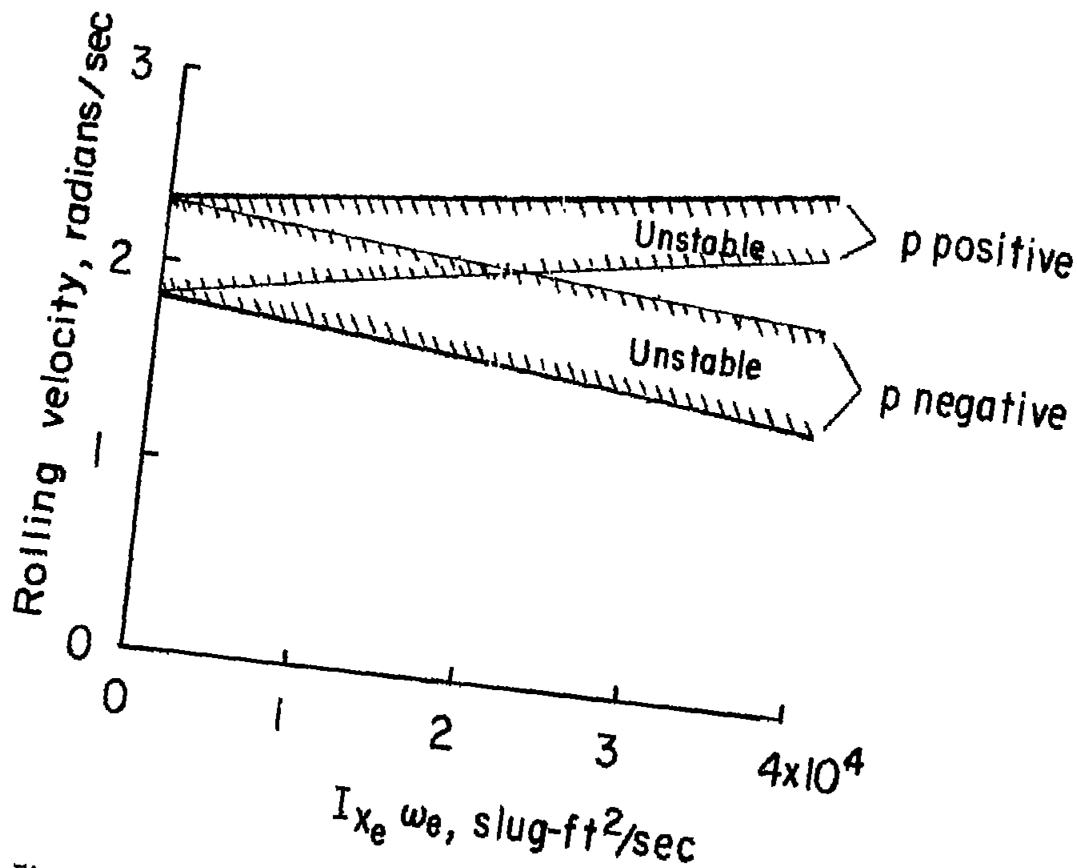


Figure 4.- Effect of engine momentum $I_{X_e}\omega_e$ on rolling-velocity range for which example aircraft is unstable.

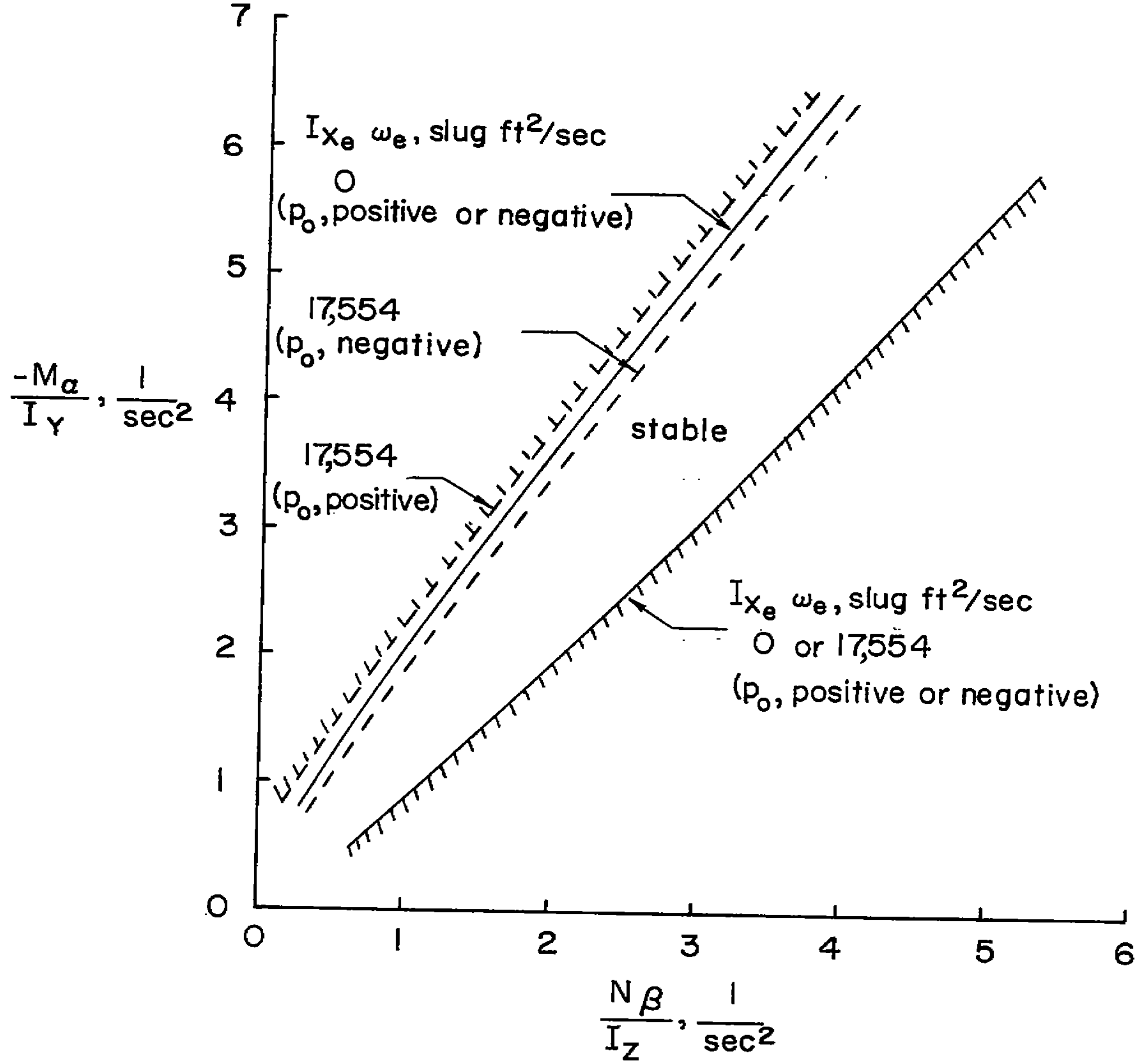


Figure 5.- Envelopes of divergence boundaries presented in figures 2 and 3.